Bayesian estimation and comparison for idiographic networks

Siepe, Kloft & Heck University of Marburg 16.07.2024

Dynamic Networks

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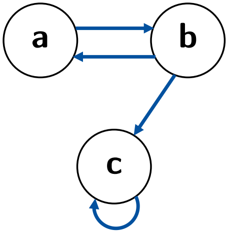
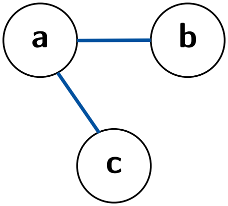
Temporal



Contemporaneous



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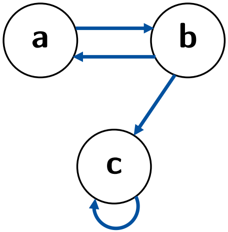

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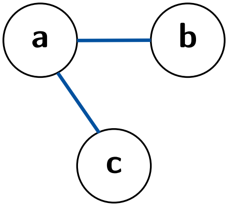

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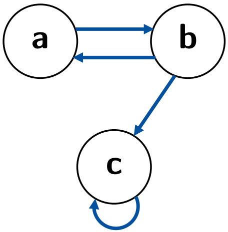

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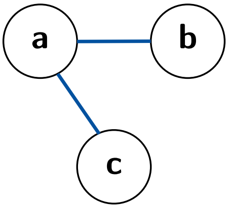

- Issues: Questionable performance in typical psychological data (Hoekstra et al., 2022; Mansueto et al., 2023)
- New ways needed to assess uncertainty

Bayesian gVAR Estimation

- Gibbs sampler in R package BGGM (Williams and Mulder, 2021)

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$$\boldsymbol{y}_t = \boldsymbol{B}\boldsymbol{y}_{t-1} + \zeta_t$$

$$\zeta_t \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Theta}^{-1})$$

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Contemporaneous Network

$$\rho_{ij} = \frac{-\Theta_{ij}}{\sqrt{\Theta_{ii}\Theta_{jj}}}.$$

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$$\rho_{ij} \sim \text{Beta}\left(\frac{\delta}{2}, \frac{\delta}{2}\right)$$

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- DGP: Real-world examples with six and eight nodes & dense network

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(Bayesian gVAR without thresholding and Bayesian gVAR with CI-based thresholding: } Wide & narrow priors)

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- 1000 Monte Carlo replications

Simulation 1: Results

Sparse true network:

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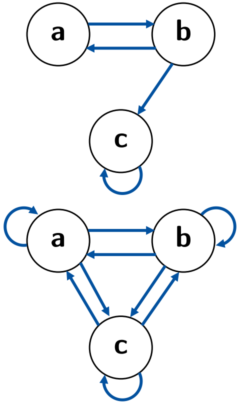

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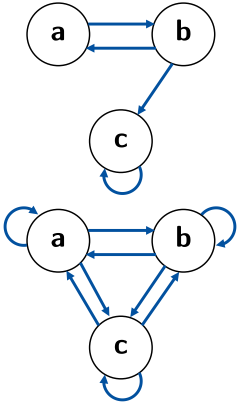

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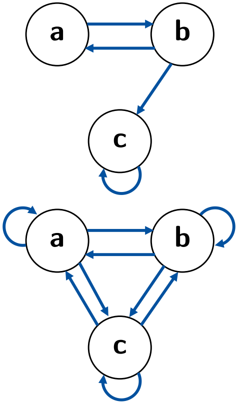

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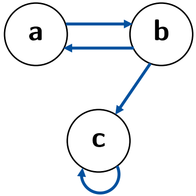
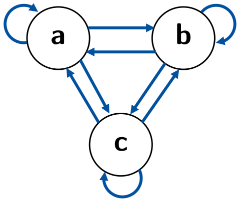
➥Choice of method depends on assumptions





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- Randomly draw matrix pairs from each posterior of two networks
  ➥ Obtain reference distribution of uncertainty
- Comparison of empirical norm with reference distribution for temporal and contemporaneous network

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- DGPs & time series length:
  Same as in first simulation

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Largest × {1.4, 1.6}

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All $\pm$ {0.05, 0.10, 0.15}

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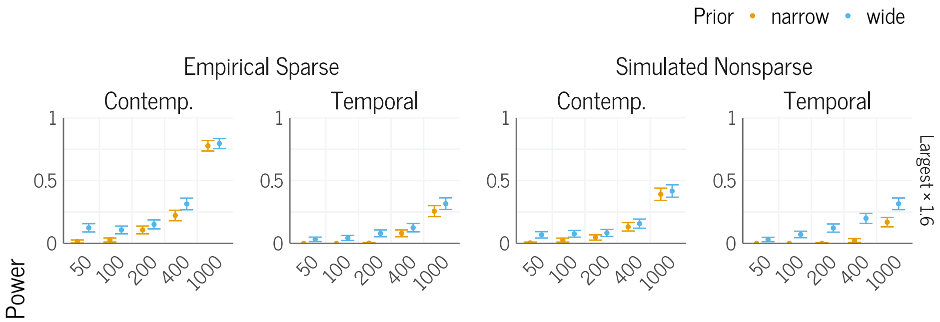

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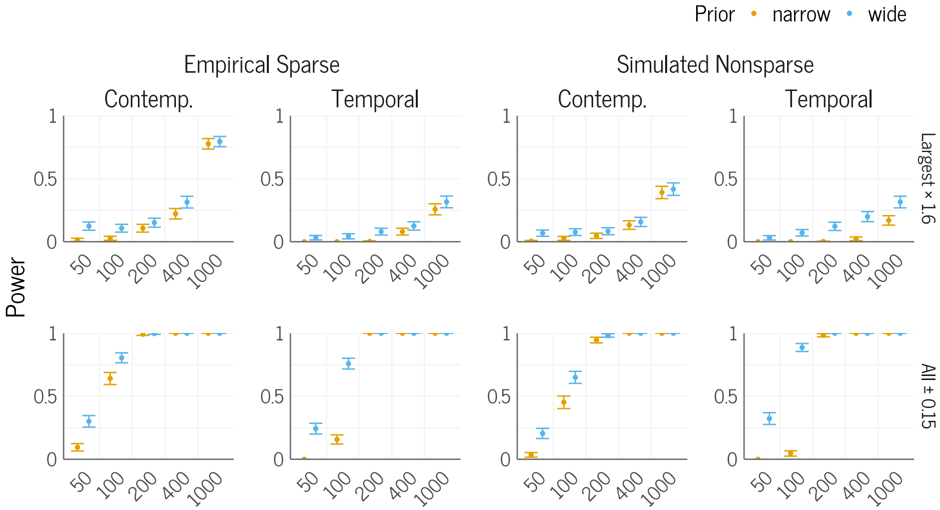
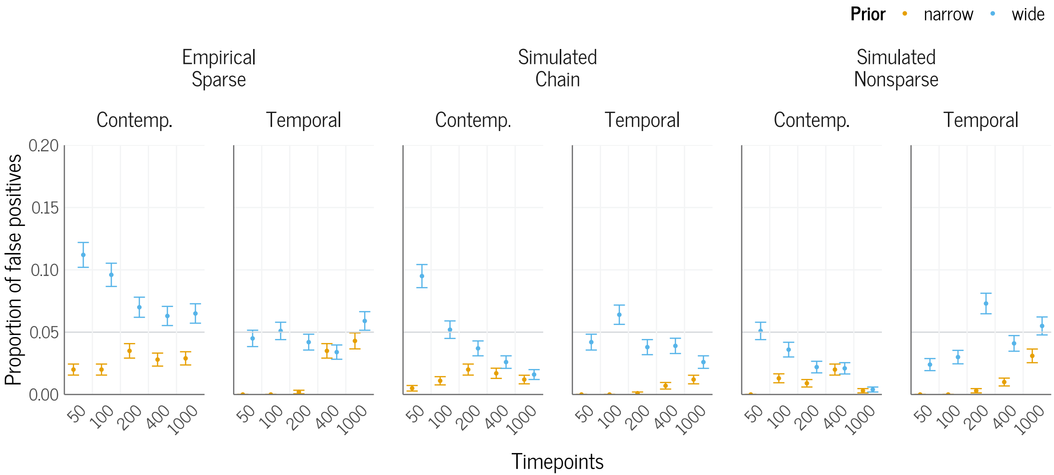


False Positive Rate



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  - ➥ Choice of estimation method depends on sparsity assumption
- The test may guard against wrong interpretations of heterogeneity
  - Implemented in R package `tsnet` (available on CRAN)
  - Potential use in intra-individual comparisons

References

Hoekstra, R. H. A., Epskamp, S., & Borsboom, D. (2022). Heterogeneity in individual network analysis: Reality or illusion? *Multivariate Behavioral Research*, *58*(4), 762–786. https://doi.org/10.1080/00273171.2022.2128020

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Get in Touch

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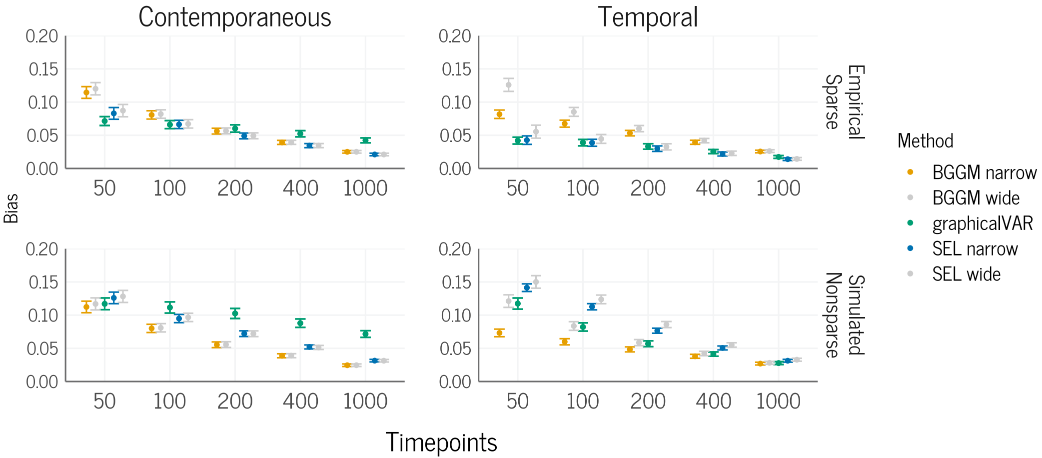
Manuscript:
psyarxiv.com/uwfjc/

Siepe, B.S., Kloft, M., Heck, D.W. (2024). *Bayesian Estimation and Comparison of Idiographic Network Models*. Psychological Methods, In Press

Backup Slides

Simulation 1 Results



Visualization of Test